



O'Donnell, M., Chenchiah, I. V., Weaver, P., & Pirrera, A. (2016). *Coupling of helical lattice structures for tunable non-linear elasticity*. Abstract from Multiscale Innovative Materials and Structures, Salerno, Italy. <http://www.multiscale.unisa.it/programme>

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# Coupling of helical lattice structures for tunable non-linear elasticity

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## Introduction

Helical lattice structures as investigated by Pirrera *et al.* [1] possess non-linear elastic responses that may be tuned to offer various distinct behaviours such as zero stiffness and multi-stability. Potential applications of such helices were explored in further detail by O'Donnell *et al.* [2] by combining the lattice with an elastic medium permitting the system to display pseudo-ductility, thus highlighting one of many possible applications. Other such applications include, for example, adaptive and deployable structures, robotics, and vibration isolators. By coupling the helical lattice with a secondary system, additional response characteristics, that cannot be achieved by the lattice alone, can be obtained. A natural extension of [1,2], explored in this paper, is to consider the effects of combining multiple concentric lattice structures, coupled through radial springs, resulting in a design space that offers significant potential for non-linear elastic tailoring.

As is often observed in biological systems, structural hierarchy offers mechanisms through which novel response characteristics may be observed [3-6]. The helices in this investigation were inspired by the virus bacteriophage T4 [1]. The composite behaviour of the system under investigation exploits such hierarchy in order to obtain highly tuned non-linear force displacement behaviour. The helices in each lattice are formed from pre-stressed composite strips, the lattices are then combined concentrically via elastic springs to form a composite helical system. An outline of the analytical modelling framework developed to capture this behaviour is now discussed.

## Energy Formulation

Pirrera *et al.* developed an energy based formulation for the helical lattice where individual helices are considered as inextensional strips that lie upon the surface of a cylinder [1]. The strips are sufficiently narrow that the behaviour across the width can be ignored. Furthermore, the hinge points in the lattice structure remain fixed such that individual helices do not slide over each other but together can translate in space. If a further condition is imposed on the helical lattice, that of reinforcing strips with a common pitch angle, there is no lattice twist under extension and the lattice's length uniquely defines its state.

The energy associated with deformations of the helix can be defined in terms of the extension of a representative unit cell [1], figure 1. For brevity the complete formulation is not presented here, instead we refer to the non-dimensional energy of a unit cell,  $\Pi_{\text{cell}}(h, [l, d_{11}, d_{16}, d_{66}, \kappa_{0x}, \kappa_{0xy}])$ , a function of the helical lattice's structural properties and lengthwise extension,  $h$ . The remaining structural parameters, the reduced bending stiffness terms,  $d_{ij}$ , as defined in CLT [7], unit-cell dimension  $l$ , and  $\kappa_0$ , the helical strips' pre-curvatures, determine the energetic landscape allowing each individual lattice to be tuned significantly.

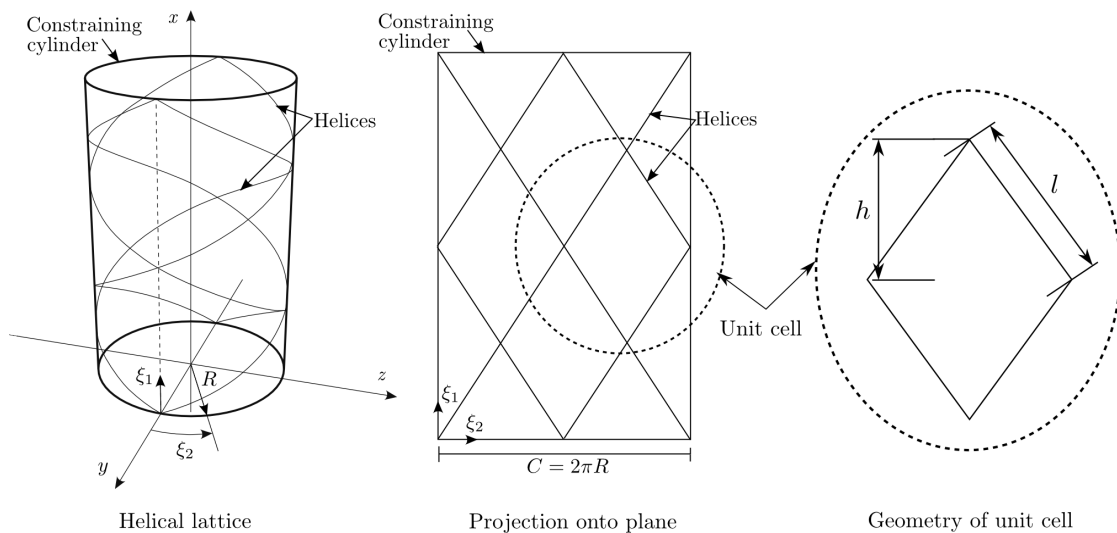


Figure 1: Geometry of a helical lattice, projection onto the plane and its representative unit cell.

For the coupled system composed of multiple lattices, the global unit cell comprises multiple unit-cells of each lattice ordered with index  $i$  from the centre outwards, figure 2. For the sake of simplicity and manufacturability we fix an integer number of longitudinal unit cells in the global unit cell having a height,  $2H$ . This therefore imposes the condition  $H = m_i l_i$ , where  $m_i$  is the longitudinal stacking number. Furthermore, to ensure that the helices are concentric, the number of unit cells in a circumferential band must satisfy,  $l_i n_i < l_{i+1} n_{i+1}$ , where  $n_i$  is the circumferential banding number of the  $i^{\text{th}}$  helix. The common extension of all the  $i^{\text{th}}$  helix's unit cells is  $h_i = H/m_i$ , thereby allowing the total energy associated with that helix to be defined as a multiple of the number of unit cells contained in the global unit,  $\Pi_i = m_i n_i \Pi_{\text{cell}_i} (H/m_i)$ , and the system's helical energy as their sum,  $\Pi_H = \sum_i \Pi_i$ .

We are free to align each of the concentric helices radially, therefore there is at least one configuration for which hinge points align thus offering a location for the coupling springs – the maximum number of joining locations is given by greatest common divisor of all radial banding numbers,  $n_i$ . For analysis purposes the multiplicity of joining points is redundant because several coupling springs between each concentric layer may instead be consolidated energetically into one effective spring of stiffness,  $k_i$ . For a given global cell extension the radius of the  $i^{\text{th}}$  helix is,  $r_i = n_i / \pi \sqrt{l_i^2 - (H/m_i)^2}$ . The energy associated with deformation of the  $i^{\text{th}}$  spring,  $\Pi_{k_i} = k_i / 2 (r_{i+1} - r_i - \gamma_i)^2$ , is then determined by the difference in radial diameter between concentric layers. This deformation is given relative to the spring's neutral position,  $\gamma_i$ . Thus the total spring energy is  $\Pi_K = \sum_i \Pi_{k_i}$ . The total energy of the coupled system can then be expressed as the total of all energetic contributions,  $\Pi = \Pi_H + \Pi_K$ . The effective force-extension relationships can be obtained directly from this energy formulation and the stability of any resulting equilibrium positions assessed. The full potential of the non-linear responses is then investigated by tuning the system's defining parameters.

## Conclusions

An energy-based model is presented for determining the response of hierarchical coupled helical lattice systems. The hierarchical structure presented offers an expanded design space beyond a single lattice thereby allowing significant potential for non-linear elastic tailoring. These structures offer a route to the development of bespoke non-linear, yet repeatable, responses for various distinct applications.

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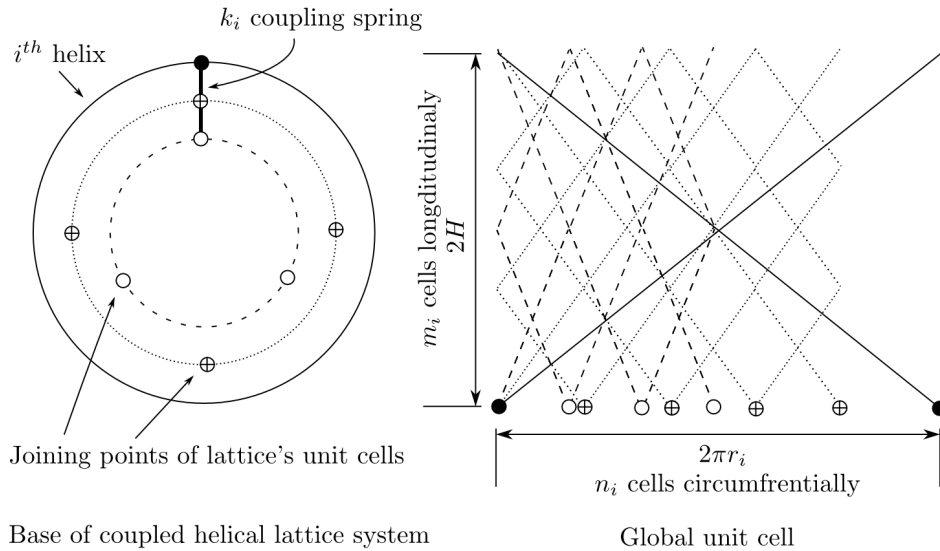


Figure 2: Layout of coupled helical system and the global unit cell composed of multiple unit cells of each helix.